Optimal planning and its limits

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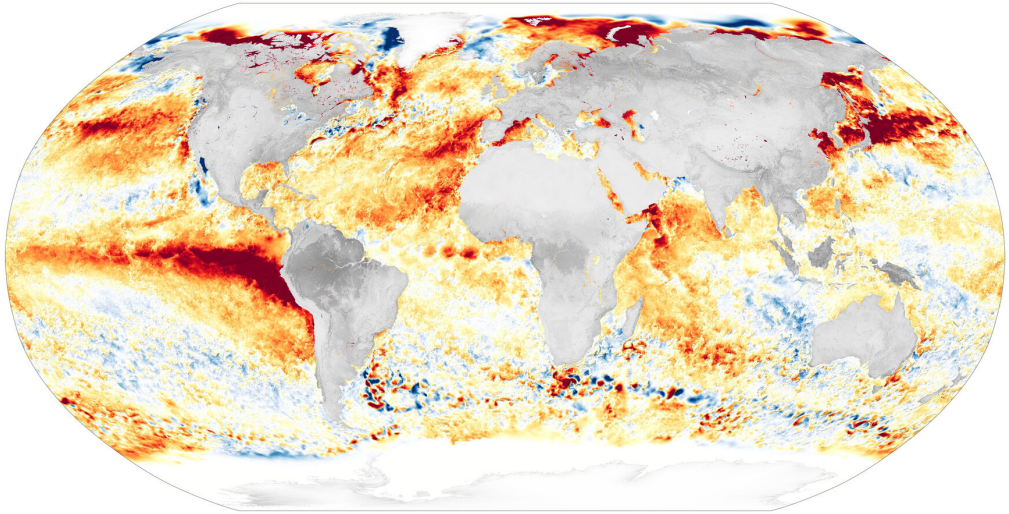
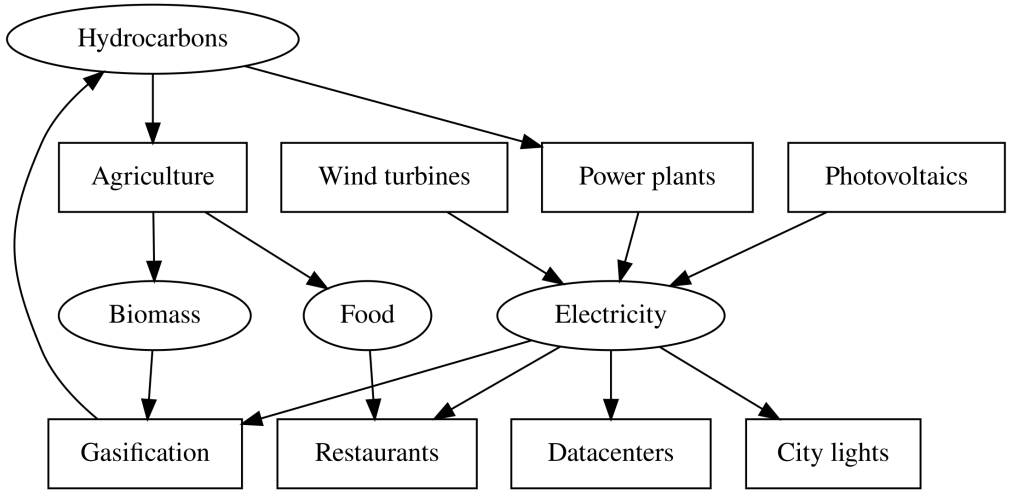
Motivation



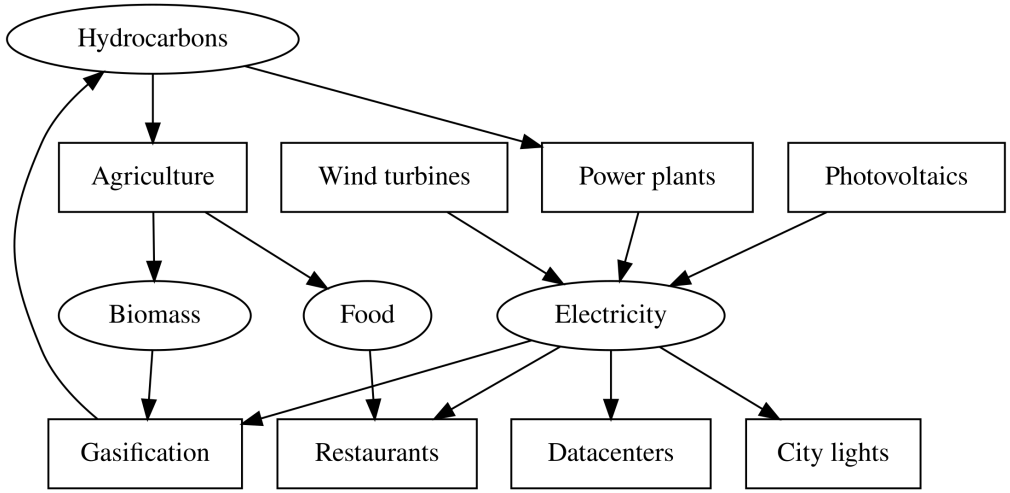
- Climate change reversal under hard physical constraints
  - Atmospheric GHG composition must be less than X by year Y
- Large investments, existing technologies
- Market allocation too slow and wasteful
- There is no alternative to planning

Linear planning



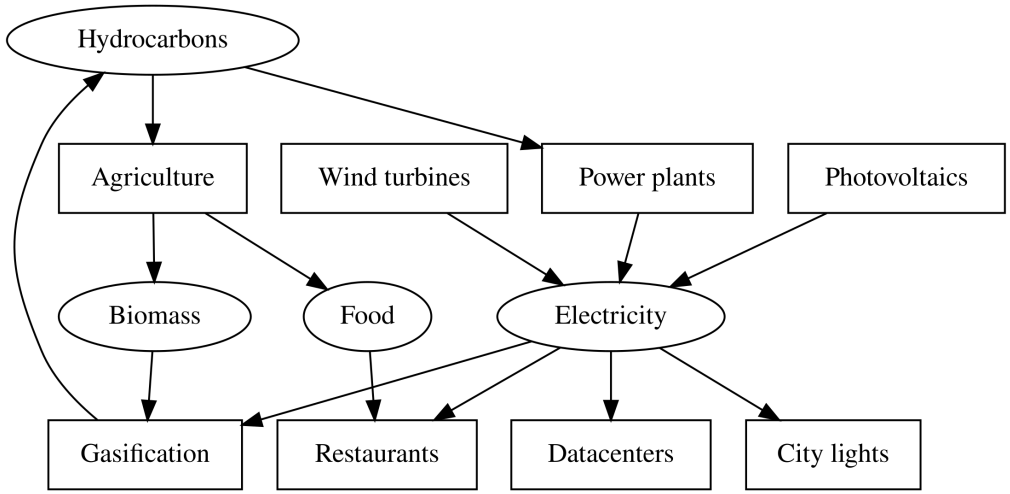
- Set of tentative production levels $\boldsymbol{x}$ across units of production and time horizon ('the plan')
- Net outputs $A\boldsymbol{x}$ when linearizing around current operating point ($\sim$ Jacobian)
- Must also satisfy constraints: $A\boldsymbol{x} \geq b$
- A and b derived from structure of economy, demand, stocks, physical constraints and politics

Linear planning



- $A = A_{\text{output}} - A_{\text{input}}$
- $b = b_{\text{consumption}} + b_{\text{investment}} + b_{\text{trade}} + b_{\text{physical}}$

Linear planning

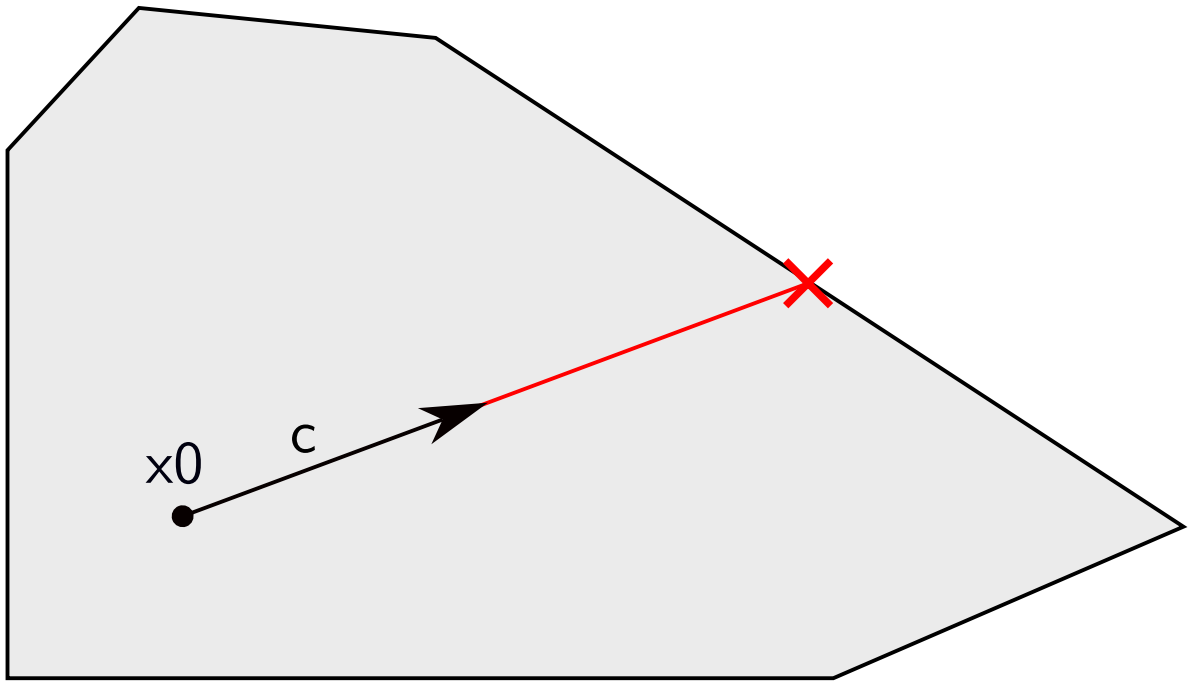


- Optimal plan minimizes/maximizes $c^T \boldsymbol{x}$ subject to $A\boldsymbol{x} \geq b$
  - Linear program
- c is a politically decided cost function (e.g. labor time)
- Plan $\boldsymbol{x}$ is broadcast to units of production
- Plans are recomputed as orders are accepted, deliveries are made, problems arise etc. (feedback)

Complexity

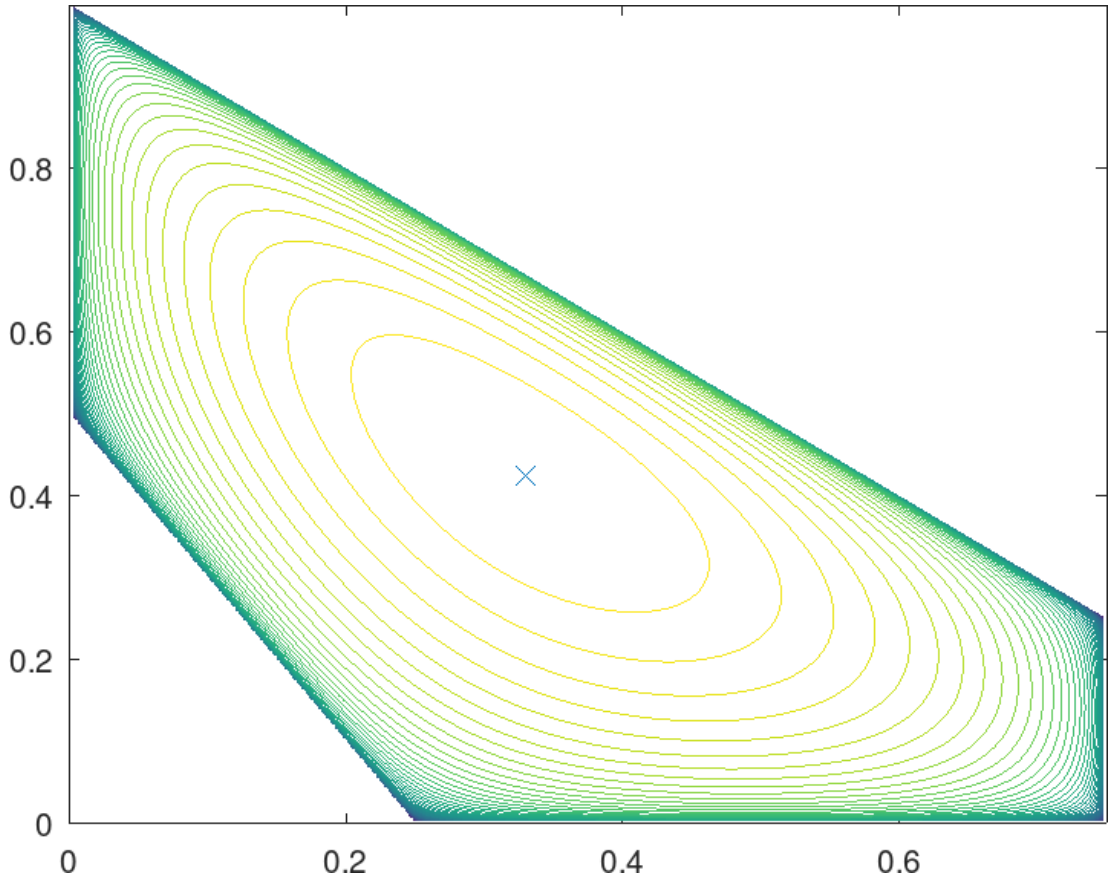
- Solving LP exactly is likely NP, while approximate solution is P
- Sparse problems are easier
- Reusing old solution is faster than solving from scratch
- Predictor-corrector methods are fast in practice
- Tens of billions of variables appear feasible
- Cluster speedup is $O(\sqrt{P})$

Relaxations



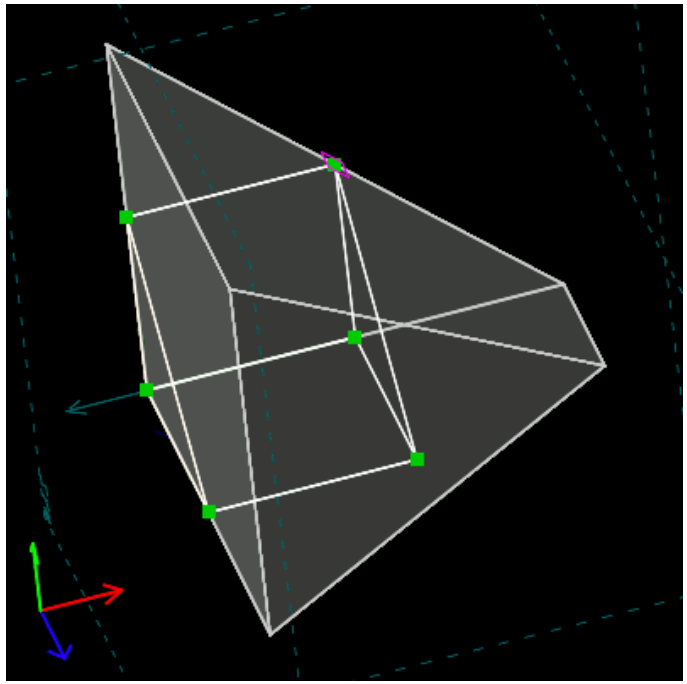
- Only trace in direction of c, stop at nearest constraint
- $O(\text{nnz}(A))$, same as sparse matrix-vector multiplication

Relaxations



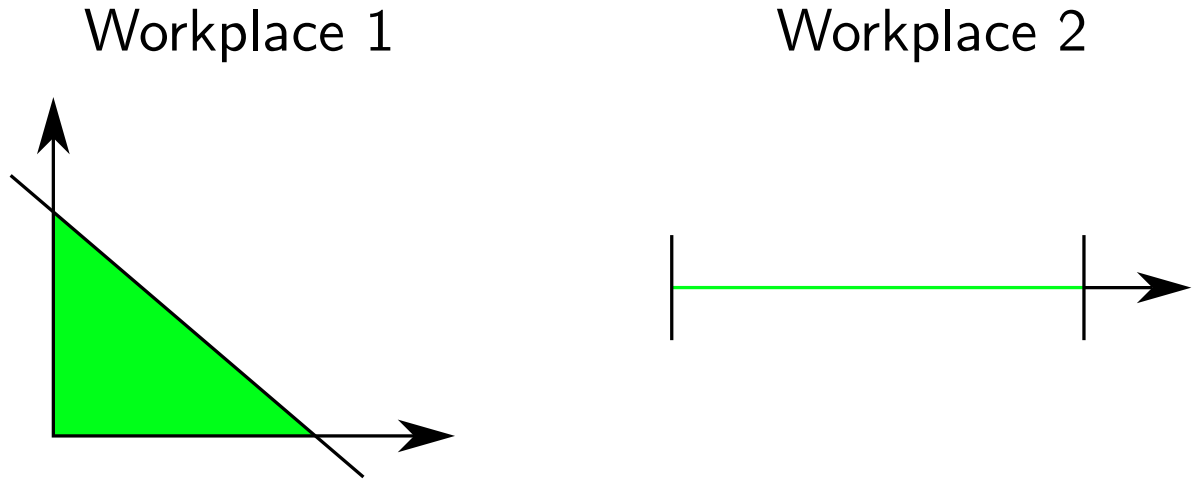
- Seek centrality rather than optimality
- Any constraint can be moved 1/3 toward the center and the system recentered in at most 206 linear system solves, conjecturally 26

Relaxations



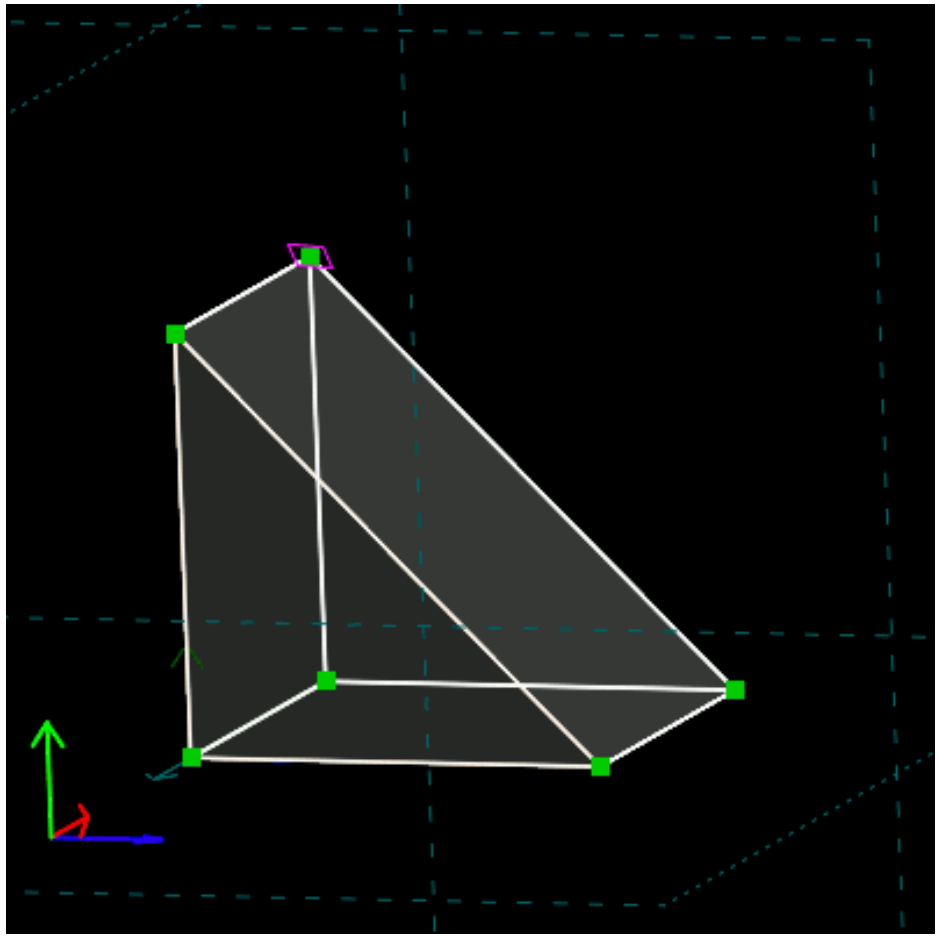
- Seek inscribed Cartesian product of workplaces/localities
- Makes localities orthogonal $\rightarrow$ autonomy
- Each locality can decide own objective function, or none
- Process can be applied recursively
- `https://www.haerdin.se/blog/2023/05/21/quantifying-autonomy-in-planning/`

Prismatic polytope



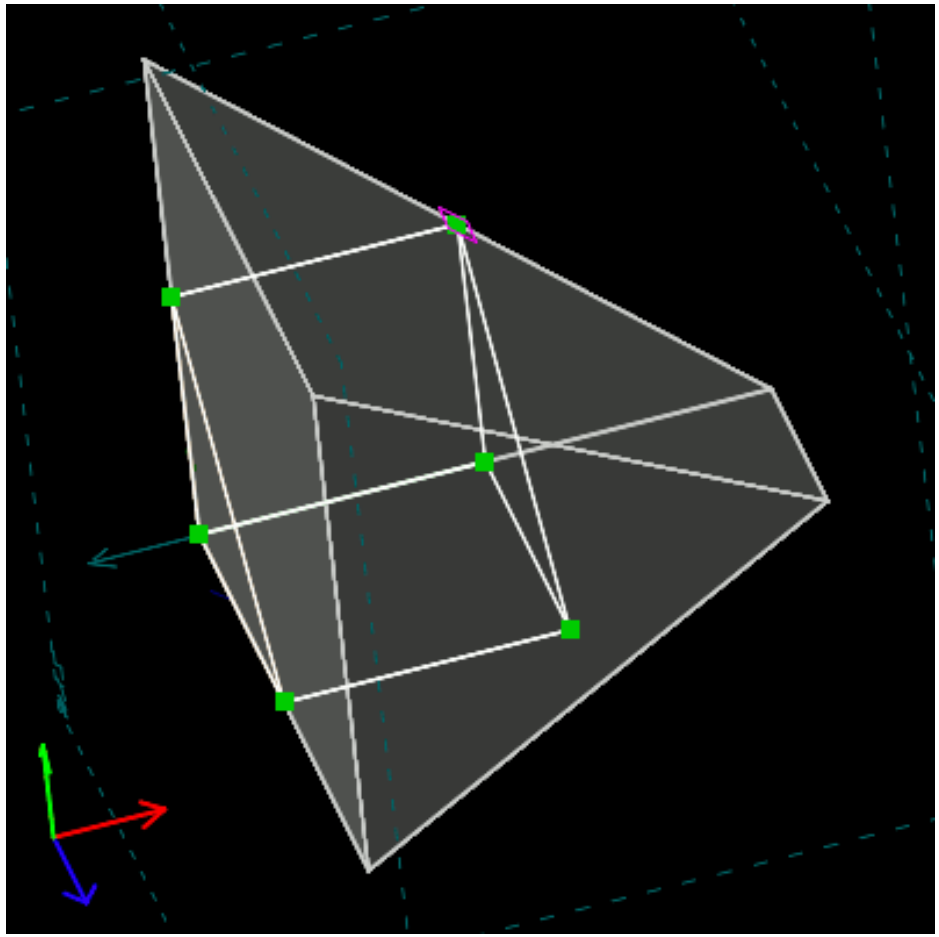
- Two workplaces
- One has two production methods plus a linear constraint
- The other just one production method with bounds

Prismatic polytope



- Cartesian product of workplaces is a prism
- Both workplaces fully autonomous

Prismatic polytope



- Prism can be feasibly inscribed in global system
- May have to shrink prism to fit, is somewhat limiting
- Autonomy is preserved

Some limitations and problems

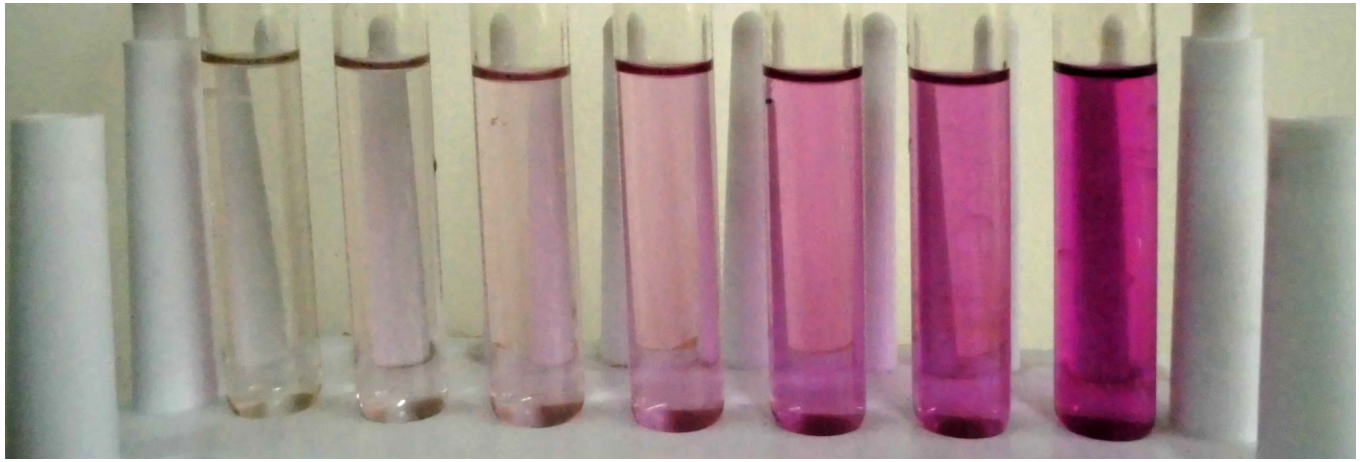
- Imperfect data
  - A and b assumed accurately known
  - How much manual data entry can we demand? Automate.
  - Statistics will likely help
- How to account for overhead, depreciation and side-products?
- Real economy is non-linear and non-convex

Information

title	release_year	length	replacement_cost
West Lion	2006	159	29.99
Virgin Daisy	2006	179	29.99
Uncut Suicides	2006	172	29.99
Tracy Cider	2006	142	29.99
Song Hedwig	2006	165	29.99
Slacker Liaisons	2006	179	29.99
Sassy Packer	2006	154	29.99
River Outlaw	2006	149	29.99

- Information is never perfect
- Some production processes can be known accurately
  - BOM for electronics
  - Chemical processes
- Some production is one-off (repairs)
- Some production is hard to predict (farming)
- Incorrect data entered accidentally or nefariously
- Labour is always uncertain

Some potential solutions



- Symmetric access to information
  - Can inspect each others' numbers → wiki magic
  - Also essential to democracy (data bunkers → tiny popes)
- Automatic checks ("gates" in Soviet parlance)
- Overstated inputs
  - Lower allocations
  - Solver routes around inefficient workplaces

Other concerns

- Some workplaces have no outputs, but they do affect b
  - Schools, hospitals etc.
- Estimating consumer demand
  - Use predictive statistics
  - Encourage pre-orders
- Remuneration
  - Labour vouchers, ration books, auctions etc.
  - Some goods and services given for free
- How do workers interact with the system? Can they say no?

What is to be done?

- Develop formalisms and theory
- Develop software
  - `https://github.com/lokehagberg/rhp`
- Popularize and disseminate ideas
- Pilot projects

The end

Questions?